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Theta angles and UV-IR matching of 't Hooft anomaly & global inconsistency

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(Based on works with Y. Kikuchi, T. Misumi, N. Sakai).

't Hooft anomaly matching

QFT with symm. G — A : background gauge field. $Z[A]$.

$$Z[A + d\theta] = Z[A]\, e^{i\mathcal{A}[\theta, A]}$$

(θ: G-gauge parameter; $\mathcal{A}$: anomaly. ⇒ RG-inv.)

Possible vacua (or massless excitations) are constrained by $\mathcal{A}$.

Classic ex.

N_f-flavor massless QCD.

Sym. $SU(N_f)_L \times SU(N_f)_R \times U(1)_V$.

⇒ 't Hooft anomaly due to ABJ anomaly.
- Chiral baryons
- SSB & WZW term for pions.

Recent developments

Anomaly matching turns out to be valid not only for chiral sym., but also for discrete sym., higher-form sym, etc.

Recent papers

- Yang-Mills theory at $\theta = \pi$ (Gaiotto, Kapustin, Komargodski, Seiberg) (⇒ even at finite T)
- Bifundamental QCD at $\theta_i = \pi$ (Tanizaki, Kikuchi)
- 2d & 3d $\mathbb{CP}^{N-1}$ model (Komargodski, Sharon, Thorngren, Zhou)
- Domain wall physics of 3d $\mathbb{CP}^{N-1}$ model (Komargodski, Sulejmanpasic, Ünsal)
- Roberge-Weiss phase transition of N-flavor massless QCD ($N_c = N_f$, at finite T) (Shimizu, Yonekura)
- QCD_4 @ $\theta = \pi$, Dashen phenomenon (Gaiotto, Komargodski, Seiberg).
- Systematic construction of anomaly with S^1 compact. under T.B.C. (Tanizaki, Misumi, Sakai) ⇒ $\mathbb{Z}_N$-twisted $\mathbb{CP}^{N-1}$, $\mathbb{Z}_N$-QCD.

Motivation Large-N limit of $SU(N)$ Yang-Mills.

$$Z(\theta) = \int \mathcal{D}A \exp\left[-\frac{1}{2g^2}\int \mathrm{tr}(G\wedge *G) + \frac{i\theta}{8\pi^2}\int \mathrm{tr}(G\wedge G)\right].$$

$$G = dA + iA\wedge A.$$

't Hooft large-N limit: $N\to\infty$ with fixed $\lambda = g^2 N$.

$$Z(\theta) = \int \mathcal{D}A \exp\left[N\left(-\frac{1}{2\lambda}\int \mathrm{tr}(G\wedge *G) + \frac{i(\theta/N)}{8\pi^2}\int \mathrm{tr}(G\wedge G)\right)\right] = e^{-V_4\cdot E(\theta)}$$

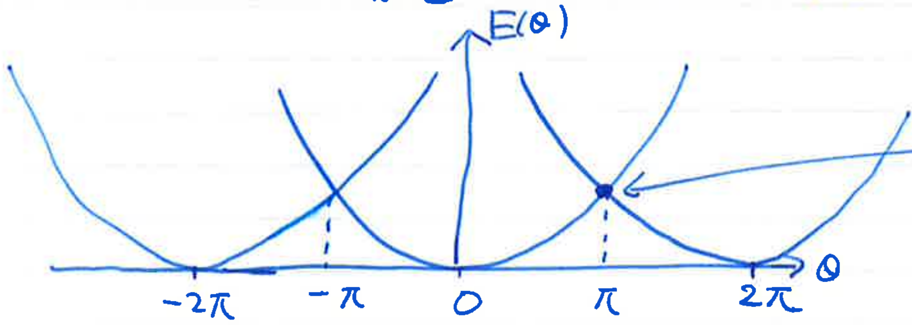
$\Rightarrow$ $\tilde{\theta} = \frac{\theta}{N}$ is a natural parameter for 't Hooft large-N limit.

$$E(\theta) = N^2 f\left(\frac{\theta}{N}\right). \qquad (f:\ \text{a smooth func.})$$

(N^2: # gluons)

However, $E(\theta + 2\pi) \neq E(\theta)$ unless f is const.

$$\Rightarrow \quad E(\theta) = \min_{k\in\mathbb{Z}}\left(N^2 f\left(\frac{\theta+2\pi k}{N}\right)\right). \qquad (\text{Witten}).$$



degenerate ground state.

CP is spontaneously broken.

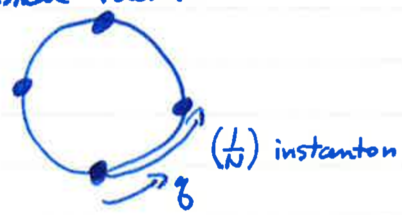
(We'll show this by anomaly matching.

Quantum Mechanics example.

$q \sim q + 2\pi$ ($\in S^1$)

$$\frac{1}{\hbar}S = \frac{1}{\hbar}\int d\tau \left\{\frac{1}{2}\left(\frac{dq}{d\tau}\right)^2 + (1-\cos(Nq))\right\} - \frac{i\theta}{2\pi}\int dq.$$

N classical vacua.



DIGA at $\hbar \ll 1$:

$$E_k(\theta) \sim -\cos\left(\frac{\theta+2\pi k}{N}\right).$$

$$E(\theta) = \min_{k=0,\cdots,N-1} E_k(\theta).$$

Symmetry of the system:

$\mathbb{Z}_N$ shift $\quad q \longrightarrow q + \frac{2\pi}{N}$. $\qquad U = e^{\frac{2\pi i}{N}\hat{p}}$.

$\mathbb{Z}_2$ at $\theta = 0, \pi$: $q \longrightarrow -q$. $\qquad C$.

Classically, $\quad U^N = 1, \quad C^2 \bar{=} 1, \quad C \cdot U \cdot C^{-1} = U^{-1}$.

QM at $\theta = \pi$: $\quad U^N = 1, \quad C^2 = 1, \quad C \cdot U \cdot C^{-1} = e^{\frac{2\pi i}{N}} U^{-1}$.

$\downarrow \qquad \downarrow$

$U\psi = e^{\frac{2\pi i}{N}k}\psi \qquad C\psi = \eta\psi \Rightarrow C \cdot U \cdot C^{-1}\psi = e^{\frac{2\pi i}{N}} U^{-1}\psi$

$\qquad \| \qquad\qquad \|$

$\qquad e^{\frac{2\pi i}{N}k}\psi \qquad e^{\frac{2\pi i}{N}} e^{-\frac{2\pi i}{N}k}\psi$.

For $\psi \neq 0$, $\quad e^{\frac{2\pi i}{N}(2k-1)} = 1$, i.e. $2k = 1 \mod N$.

If N is even, this is impossible $\Rightarrow$ No singlet state.
($\cong$ Kramers doublets).

<u>"Modern" derivation</u>

Gauge the $\mathbb{Z}_N$ shift symm. $q \mapsto q + \frac{2\pi}{N}$.

To do it, introduce the "$\mathbb{Z}_N$-gauge field" A:

$\mathbb{Z}_N$ gauge field $:=$ $U(1)$ gauge field A with the constraint
$$NA = d\phi.$$

(One can regard this as a LEFT for Higgsing.
$z \in \mathbb{C}$ with charge $-N$.
$$\mathcal{L} = |(d + iNA)z|^2 + V(|z|^2).$$
Higgsing $\langle |z|^2 \rangle \neq 0$. $\Rightarrow z \simeq \upsilon e^{i\phi}$.
$$\mathcal{L}_{\text{LEFT}} = \upsilon^2 |(d\phi + NA)|^2. \xrightarrow{\upsilon \to \infty} \delta(d\phi + NA).$$)

1-dim. topological $\mathbb{Z}_N$ theory : $CS = k\int A \quad (k \in \mathbb{Z}_N)$.

We couple our system to the above top. theory :
(Minimal coupling).

$$\begin{cases} dq \Rightarrow dq + A. \\ \cos(Nq) \Rightarrow \cos(Nq+\phi). \end{cases}$$

$$\frac{1}{\hbar} S_{\text{gauged}} = \frac{1}{\hbar}\int \left\{\frac{1}{2}|dq+A|^2 + (1-\cos(Nq+\phi))\right\} d\tau - \frac{i\theta}{2\pi}\int (dq+A).$$

"$\mathbb{Z}_N$ gauge invariance"

$$q \to q+\lambda, \quad A \to A - d\lambda, \quad \phi \to \phi - N\lambda.$$

Therefore,

$$Z_\theta[A] = \int \mathcal{D}q \; e^{-\frac{1}{\hbar}\int d\tau \left\{\frac{1}{2}|dq+A|^2 + (1-\cos(Nq+\phi))\right\} + \frac{i\theta}{2\pi}\int(dq+A)}.$$

We obtain

$$Z_{\theta=\pi}[C\cdot A] = Z_{\theta=\pi}[A]\, e^{-i\int A}.$$

't Hooft anomaly. $\Rightarrow$ $e^{-\frac{2\pi i}{N}k}$ $(k = 0, 1, \cdots, N-1)$

$\Rightarrow$ phase in the comm. rel. $CUC^{-1}U^{-1} = e^{\frac{2\pi i}{N}}$.

(Even N: No possible counter term. Odd N: possible counterterm $= \frac{N+1}{2}\int A$ $\to$ inconsistent with the CP-inv. counterterm @ $\theta = 0$.)

SU(N) YM @ $\theta = \pi$

SU(N) YM theory has the following symmetry

- Center symmetry ($\mathbb{Z}_N$ one-form symmetry).

  Wilson line $W(C) = \mathrm{tr}\left(\mathcal{P}\exp\left(i\int_C a\right)\right) \mapsto e^{\frac{2\pi i}{N}} W(C)$.

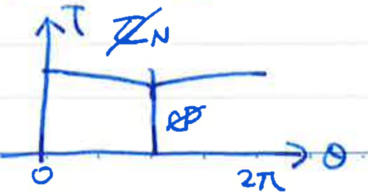
  (fields in adj. rep. do not react under this trans.).

  $\longrightarrow$ $\mathbb{Z}_N$ two-form gauge field B.

- CP at $\theta = 0, \pi$

$$Z_{\theta=\pi}[CP\cdot B] = Z_{\theta=\pi}[B]\, e^{\frac{iN}{4\pi}\int B\wedge B}.$$

anomaly



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- N-flavor massless QCD ($N_c = N_f = N$) with twisted B.C.

$$S = \frac{1}{2g^2}\int tr(G\wedge * G) + \int tr\{\bar{\Psi}\gamma^\mu(\partial_\mu + i a_\mu)\Psi\}. \qquad (\Psi = (\overset{u}{q_1}, \overset{d}{\cdots}, \overset{s\,(N=3)}{q_N}))$$

Symmetry : $\frac{SU(N)_V}{(\mathbb{Z}_N)_{\text{color-flavor}}}$ (— A, — B), $(\mathbb{Z}_{2N})$ axial.

$$Z[(A,B)] \xrightarrow{(\mathbb{Z}_{2N})_{axial}} Z[(A,B)]\, e^{\frac{iN}{4\pi}\int B\wedge B}.$$

Finite temp. with twisted B.C.

$$q_n(x, x_4 + \beta) = q_n(x, x_4)\, e^{\frac{2\pi i}{N}n + \beta\mu}.$$

($\frac{2\pi i}{N}n$: flavor-dep. imaginary chemical potential; $\beta\mu$: ← quark chemical potential)

Symmetry $(\mathbb{Z}_N)_{\text{shift}}$ — $B^{(1)}$, $q_n \mapsto q_{n+1}$, $\Phi \to e^{\frac{2\pi i}{N}}\Phi$; $\frac{U(1)^{N-1}}{(\mathbb{Z}_N)_{\text{color-flavor}}}$ (— A', — $B^{(2)}$), $(\mathbb{Z}_{2N})$ axial.

$$B = B^{(2)} + B^{(1)}\wedge \beta dx^4$$

Thus,

$$Z_{3D}[(A', B^{(1)}, B^{(2)})] \xrightarrow{(\mathbb{Z}_{2N})_{axial}} Z_{3D}[(A', B^{(1)}, B^{(2)})]\, \underbrace{e^{2\cdot\frac{iN}{2\pi}\int B^{(1)}\wedge B^{(2)}}}_{\text{'t Hooft anomaly of } \mathbb{Z}_N\text{-QCD}}.$$

$\Rightarrow$ No trivial gapped phase of $\mathbb{Z}_N$-QCD.

Conclusions

- Applicability of 't Hooft anomaly matching is nowadays extended.
- Many examples (wo. SUSY) are now studied.
- Sometimes, we can get a deeper understanding of physics.
- Sometimes, we can get a new nontrivial result.